Composing Thermostatically Controlled Loads to Determine the Reliability against Blackouts

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Agenda I

Thermostatically Controlled Loads (TCL)

- A thermostat controls the air conditioner in a house.
- The set temperature θ_s is 20 °C, the ambient temperature θ_a is 32 °C.
- The hysteresis δ is 0.5 °C.

Figure: Hysteresis

-

- The following equation describes the thermostat switching on and off:

$$\theta(t+1) = \underbrace{a\theta(t)}_{i)} + \underbrace{(1-a)(\theta_a - m(t)R \cdot P)}_{ii)} + \underbrace{g(t)}_{iii)} \; [Callaway2009, p.8] \quad (1)$$

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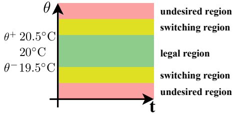
- 

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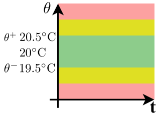

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Switch m

$$m_i(t_{n+1}) = \begin{cases} 0, & \theta(t) < \theta_s - \delta = \theta_- \\ 1, & \theta(t) > \theta_s + \delta = \theta_+ \\ m(t) & \text{otherwise} \end{cases} \quad (2)$$

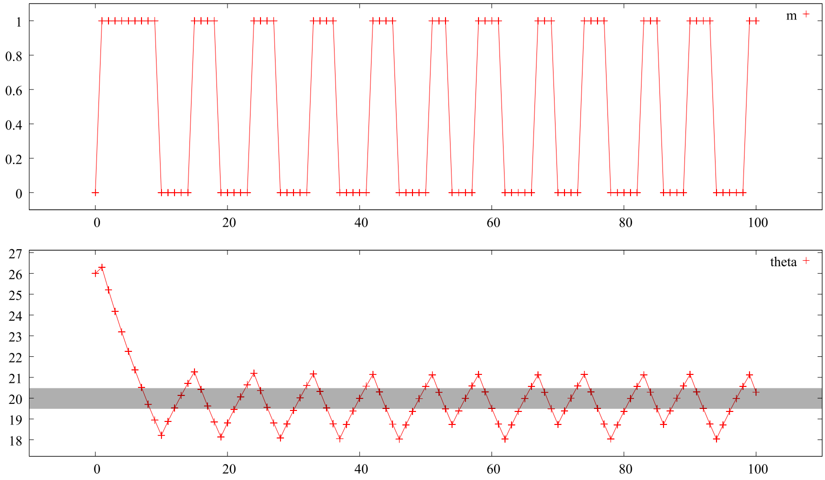
Basic Parameters

Parameter	Meaning	Standard value	Unit
R	average thermal resistance	2	$^\circ\mathrm{C}/kW$
C	average thermal capacitance	10	$kWh/^\circ\mathrm{C}$
P	average energy transfer rate	14	kw
η	load efficiency	2.5	
θ_s	temperature set point	20	$^\circ\mathrm{C}$
δ	thermostat hysteresis	0.5	$^\circ\mathrm{C}$
θ_a	ambient temperature	32	$^\circ\mathrm{C}$

Table: Model parameters [Callaway2009]

Execution Trace

. . . with basic parameters and without noise:



Basic Parameters

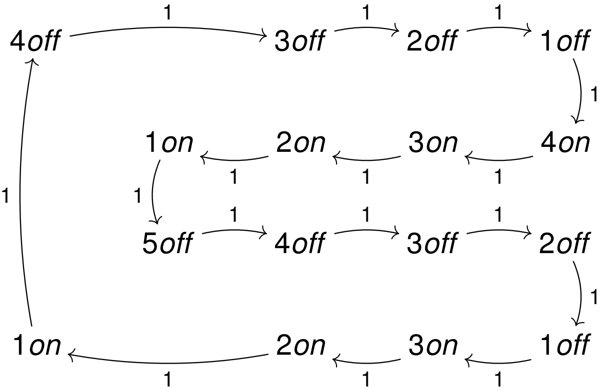
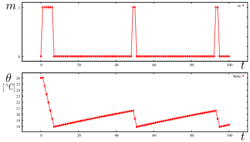
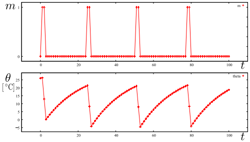


Figure: Repetitive cycle of a switch remaining in a certain state

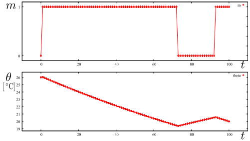
Manipulating the Parameters



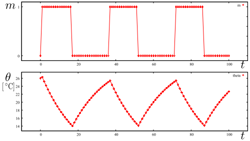
(a) $R = 20\,^{\circ}\mathrm{C} = kW$



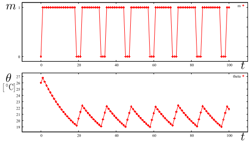
(b) $P = 140kw$



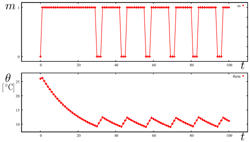
(c) $C = 100kWh/\,^{\circ}\mathrm{C}$



(d) $\delta = 5\,^{\circ}\mathrm{C}$



(e) $\theta_a = 42\,^{\circ}\mathrm{C}$



(f) $\theta_s = 10\,^{\circ}\mathrm{C}$

Binning & Noise

- Binning: discretize continuous temperature domain
- noise: add noise function (third part of equation)

$\Rightarrow$ transition probabilities to *hop* from one bin to the next bin within one discrete time step.

Simulating Temperature Evolution

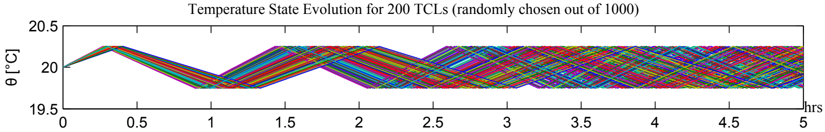


Figure: Simulating Temperature Evolution, [Koch2009]

Binned Transition Abstraction

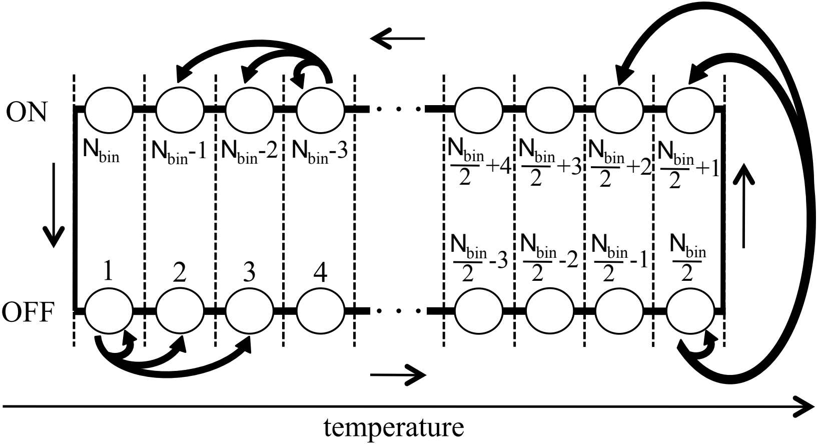


Figure: Binned Transition Abstraction, [Koch2009]

From BMP to DTMC

↓ from/to →	1 on	2 on	1 off	2 off
1 on	p_1	$1 - p_1$		
2 on		p_2	$1 - p_2$	
1 off			p_3	$1 - p_3$
2 off	$1 - p_4$			p_4

Table: Example symbolic DTMC for a surrogate housing $\mathcal{D}_1$

16 states, 64 transitions → ten states, 36 transitions

Two Houses, two Bins

↓first quarter row	first quarter column			
↓ from/to →	⟨1on, 1on⟩	⟨1on, 2on⟩	⟨1on, 1off⟩	⟨1on, 2off⟩
⟨1on, 1on⟩	p_1^2	$p_1 \cdot (1 - p_1)$		
⟨1on, 2on⟩		$p_1 \cdot p_2$	$p_1 \cdot (1 - p_2)$	
⟨1on, 1off⟩			$p_1 \cdot p_3$	$p_1 \cdot (1 - p_3)$
⟨1on, 2off⟩	$p_1 \cdot (1 - p_4)$			$p_1 \cdot p_4$

↓second quarter row	second quarter column			
↓ from/to →	⟨2on, 1on⟩	⟨2on, 2on⟩	⟨2on, 1off⟩	⟨2on, 2off⟩
⟨2on, 1on⟩	$p_2 \cdot p_1$	$p_2 \cdot (1 - p_1)$		
⟨2on, 2on⟩		p_2^2	$(1 - p_2) \cdot p_2$	
⟨2on, 1off⟩			$p_2 \cdot p_3$	$p_2 \cdot (1 - p_3)$
⟨2on, 2off⟩	$p_2 \cdot (1 - p_4)$			$p_2 \cdot p_4$

↓third quarter row	third quarter column			
↓ from/to →	⟨1off, 1on⟩	⟨1off, 2on⟩	⟨1off, 1off⟩	⟨1off, 2off⟩
⟨1off, 1on⟩	$p_3 \cdot p_1$	$p_3 \cdot (1 - p_1)$		
⟨1off, 2on⟩		$p_3 \cdot p_2$	$p_3 \cdot (1 - p_2)$	
⟨1off, 1off⟩			p_3^2	$p_3 \cdot (1 - p_3)$
⟨1off, 2off⟩	$p_3 \cdot (1 - p_4)$			$p_3 \cdot p_4$

↓fourth quarter row	fourth quarter column			
↓ from/to →	⟨1on, 1on⟩	⟨1on, 2on⟩	⟨1on, 1off⟩	⟨1on, 2off⟩
⟨2off, 1on⟩	$(1 - p_4) \cdot p_1$	$(1 - p_4) \cdot (1 - p_1)$		
⟨2off, 2on⟩		$(1 - p_4) \cdot p_2$	$(1 - p_4) \cdot (1 - p_2)$	
⟨2off, 1off⟩			$(1 - p_4) \cdot p_3$	$(1 - p_4) \cdot (1 - p_3)$
⟨2off, 2off⟩	$(1 - p_4)^2$			$(1 - p_4) \cdot p_4$

↓first quarter row	second quarter column			
↓ from/to →	⟨2on, 1on⟩	⟨2on, 2on⟩	⟨2on, 1off⟩	⟨2on, 2off⟩
⟨1on, 1on⟩	$p_1 \cdot (1 - p_1)$	$(1 - p_1)^2$		
⟨1on, 2on⟩		$(1 - p_1) \cdot p_2$	$(1 - p_1) \cdot (1 - p_2)$	
⟨1on, 1off⟩			$(1 - p_1) \cdot p_3$	$(1 - p_1) \cdot (1 - p_3)$
⟨1on, 2off⟩	$(1 - p_1) \cdot (1 - p_4)$			$(1 - p_1) \cdot p_4$

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↓ from/to →	⟨1off, 1on⟩	⟨1off, 2on⟩	⟨1off, 1off⟩	⟨1off, 2off⟩
⟨2on, 1on⟩	$(1 - p_2) \cdot_1$	$(1 - p_2) \cdot (1 - p_1)$		
⟨2on, 2on⟩		$p_2 \cdot (1 - p_2)$	$(1 - p_2)^2$	
⟨2on, 1off⟩			$(1 - p_2) \cdot p_3$	$(1 - p_2) \cdot (1 - p_3)$
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⟨1off, 2on⟩		$(1 - p_3) \cdot p_2$	$(1 - p_3) \cdot (1 - p_2)$	
⟨1off, 1off⟩			$(1 - p_3) \cdot p_3$	$(1 - p_3)^2$
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↓fourth quarter row	first quarter column			
↓ from/to →	⟨2off, 1on⟩	⟨2off, 2on⟩	⟨2off, 1off⟩	⟨2off, 2off⟩
⟨2off, 1on⟩	$p_4 \cdot p_1$	$p_4 \cdot (1 - p_1)$		
⟨2off, 2on⟩		$p_4 \cdot p_2$	$p_4 \cdot (1 - p_2)$	
⟨2off, 1off⟩			$p_4 \cdot p_3$	$p_4 \cdot (1 - p_3)$
⟨2off, 2off⟩	$p_4 \cdot (1 - p_4)$			p_4^2

Table: Example TCL DTMC composition $\mathcal{D}_2$, 16 states, 64 transitions

Wireless Sensor Network

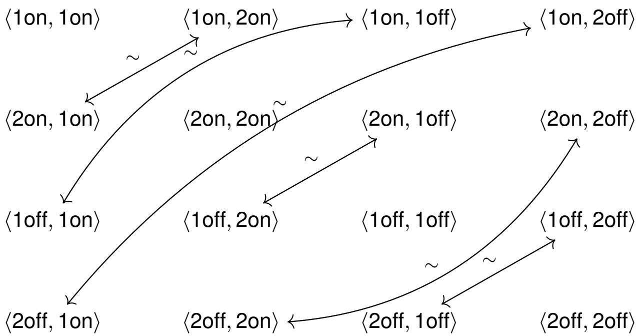


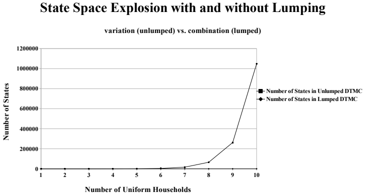
Figure: Lumping scheme showing which states are bisimilar

↓ from/to →	⟨1on, 1on⟩	⟨𝟙on, 𝟚on⟩	⟨𝟙on, 𝟙off⟩	⟨𝟙on, 𝟚off⟩	⟨2on, 2on⟩	⟨𝟚on, 𝟙off⟩	⟨𝟚on, 𝟚off⟩
⟨1on, 1on⟩	p_1^2	$2 \cdot p_1 \cdot (1-p_1)$			$(1-p_1)^2$		
⟨𝟙on, 𝟚on⟩		$p_1 \cdot p_2$	$p_1 \cdot (1-p_2)$		$(1-p_1) \cdot p_2$	$(1-p_1) \cdot (1-p_2)$	
⟨𝟙on, 𝟙off⟩			$p_1 \cdot p_3$	$p_1 \cdot (1-p_3)$		$(1-p_1) \cdot p_3$	$(1-p_1) \cdot (1-p_3)$
⟨𝟙on, 𝟚off⟩	$p_1 \cdot (1-p_4)$	$(1-p_1) \cdot (1-p_4)$		$p_1 \cdot p_4$			$(1-p_1) \cdot p_4$

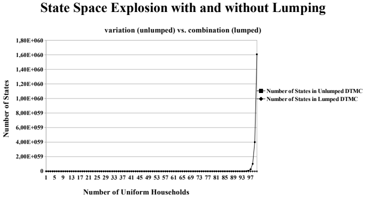
↓ from/to →	⟨𝟙on, 𝟚on⟩	⟨𝟙on, 𝟙off⟩	⟨𝟙on, 𝟚off⟩	⟨2on, 2on⟩	⟨𝟚on, 𝟙off⟩	⟨𝟚on, 𝟚off⟩	⟨1off, 1off⟩	⟨𝟙off, 𝟚off⟩
⟨2on, 2on⟩				p_2^2	$2 \cdot (1-p_2) \cdot p_2$		$(1-p_2)^2$	
⟨𝟚on, 𝟙off⟩		$p_2 \cdot p_3$	$p - 2 \cdot (1-p_3)$			$(1-p_2) \cdot (1-p_3)$	$(1-p_2) \cdot p_3$	
⟨𝟚on, 𝟚off⟩	$p_1 \cdot (1-p_4)$	$(1-p_1) \cdot (1-p_4)$				$p_2 \cdot p_4$		$(1-p_2) \cdot p_4$

↓ from/to →	⟨1on, 1on⟩	⟨𝟙on, 𝟙off⟩	⟨𝟙on, 𝟚off⟩	⟨1off, 1off⟩	⟨𝟙off, 𝟚off⟩	⟨2off, 2off⟩
⟨1off, 1off⟩				p_3^2	$2 \cdot (1-p_3) \cdot p_3$	$(1-p_3)^2$
⟨𝟙off, 𝟚off⟩		$p_3 \cdot (1-p_4)$	$(1-p_3) \cdot (1-p_4)$		$p_3 \cdot p_4$	$(1-p_3) \cdot p_4$
⟨2off, 2off⟩	$(1-p_4)^2$		$2 \cdot (1-p_4) \cdot p_4$			p_4^2

Table: Lumped DTMC $\mathcal{D}_2'$, ten states, 36 transitions



(a) Dampening the state space explosion in the first ten steps



(b) Dampening the state space explosion in the first 100 steps

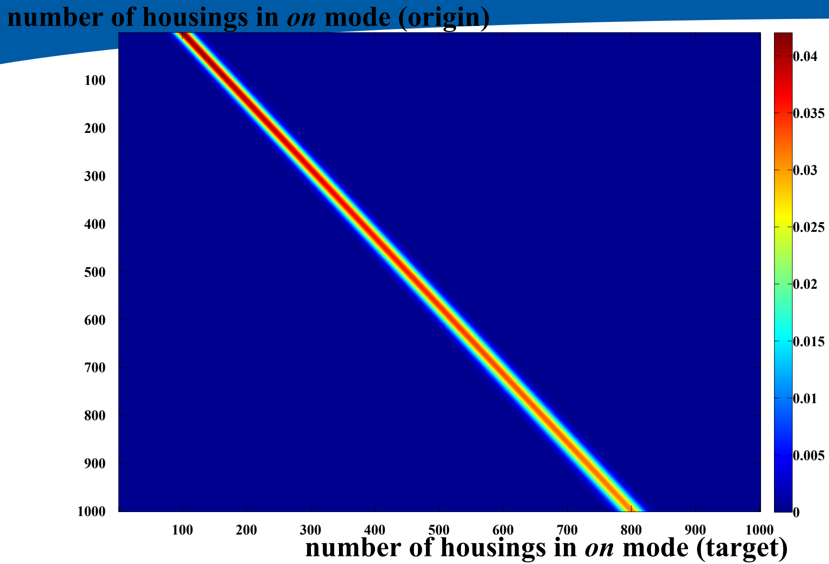


Figure: 1000 housings TCL power grid

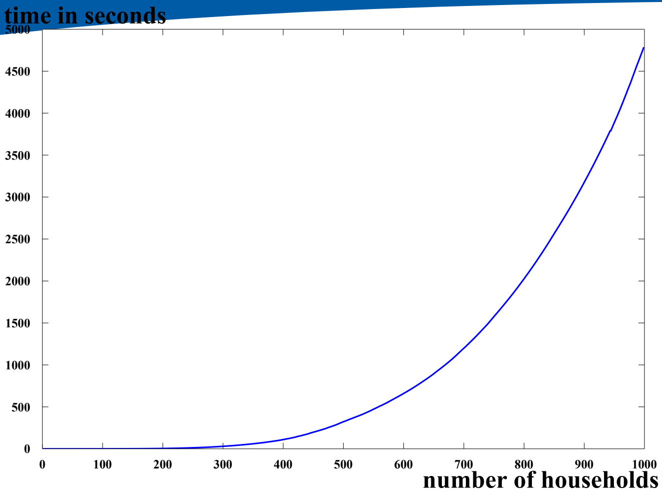
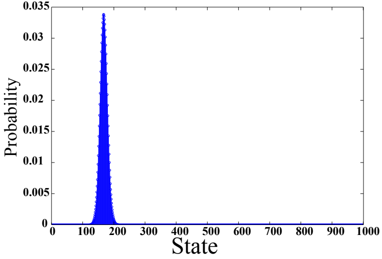
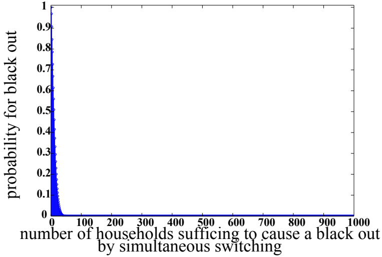


Figure: Time consumption to compute 1000 housings TCL power grid



(a) Stationary distribution



(b) Risk per time step to black out depending on the number of simultaneously switching households

Figure: Determining the risk to crash

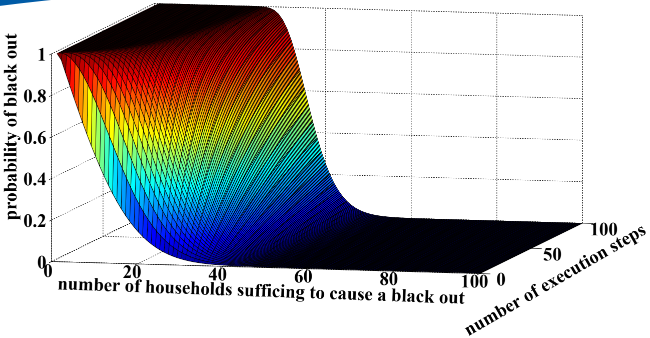


Figure: Limiting window reliability over 100 time steps

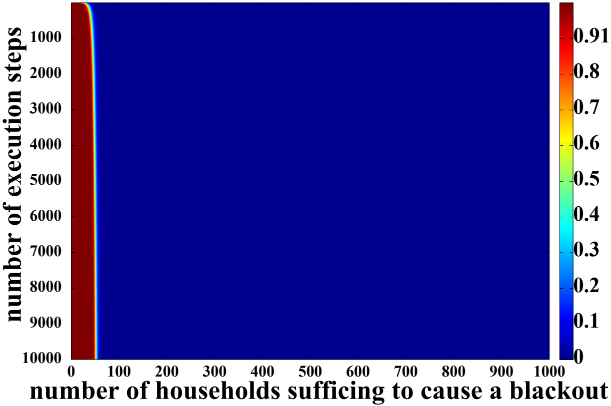


Figure: Limiting window reliability over 10,000 time steps

- **scope**: computing reliability of homogeneous TCLs against blackout
- **method**: parallel composition
- **focus**: exploit leverage, i.e. mutual independence of TCL
- **result**: compared to hierarchical/semi-hierarchical scenarios, composing mutually independent TCL is trivial

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Questions?